6 We Distribute, Yet Things Multiply

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Q1. Observe the multiplication grid below. Each number inside the grid is formed by multiplying two numbers. If the middle number of a 3 × 3 frame is given by the expression pq, as shown in the figure, write the expressions for the other numbers in the grid.

x	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

3 × 5	3 × 6	3 × 7
4 × 5	4 × 6	4 × 7
5 × 5	5 × 6	5 × 7

	pq	

Solution:
The middle element is pq. The surrounding 3 × 3 grid corresponds to factors:
- rows: p – 1, p, p + 1
- columns: q – 1, q, q + 1

Therefore:

(p – 1)(q – 1)	(p – 1)q	(p – 1)(q + 1)
p(q – 1)	pq	p(q + 1)
(p + 1)(q – 1)	(p + 1)q	(p + 1)(q + 1)

Q2. Expand the following products.
(i) (3 + u) (v – 3)
Ans:
Multiply each term:
(3+u)(v−3)
=3(v−3)+u(v−3)
=3v−9+uv−3u

(ii) 2/3 (15 + 6a)
Ans:
Distribute 2/3

= 2/3 (15+6a)
= 2/3 (15) + 2/3(6a)
=10+4a

(iii) (10a + b) (10c + d)

Ans:

Multiply all four pairs:

$=10a(10c)+10a(d)+b(10c)+bd$

$=100ac+10ad+10bc+bd$

(iv) (3 – x) (x – 6)

Ans:

$(3-x)(x-6)$

$=3(x-6)-x(x-6)$

$=3x-18-x^2+6x$

$=-x^2+9x-18$

(v) (–5a + b) (c + d)

Ans:

$=(-5a)c+(-5a)d+bc+bd$

$=-5ac-5ad+bc+bd$

(vi) (5 + z) (y + 9)

Ans:

$(5+z)(y+9)$

$=5y+45+zy+9z$

Q3. Find 3 examples where the product of two numbers remains unchanged when one of them is increased by 2 and the other is decreased by 4.

Ans:

Let the original numbers be a and b.

After the changes, the numbers become: $a+2$ and $b-4$

For the product to remain unchanged: $(a+2)(b-4)=ab$

Expand: $ab-4a+2b-8=ab$

Cancel ab from both sides:

$-4a+2b-8=0$

$2b=4a+8$

$b=2a+4$

Example 1:

Take: $a=1, b=6$

Original product: $1\times6=6$

New numbers: $1+2 = 3$, $6-4 = 2$

New product: $3 \times 2 = 6$

Example 2:

Take: $a=2$,$b=8$

Original:$2\times8=16$

New numbers:4, 4

New product:$4 \times 4=16$

Example 3:

Take: $a=3,b=10$

Original: $3\times10=30$

New numbers: 5,6

New product: $5\times6=30$

Q4. Expand

(i) $(a + ab – 3b^2) (4 + b)$

Ans:

Multiply every term of the first bracket by both terms of the second bracket:

$(a+ab−3b^2)(4+b)$

$=a(4+b)+ab(4+b)−3b^2(4+b)$

Now expand: $=4a+ab+4ab+ab^2−12b^2−3b^3$

Combine like terms: $ab+4ab=5ab$

Therefore, $4a+5ab+ab^2−12b^2−3b^3$

(ii) $(4y + 7) (y + 11z – 3)$

Ans:

Multiply 4y by every term:

$4y (y+11z−3)$

$=4y^2+44yz−12y$

Now multiply 7 by every term:

$7(y+11z−3)$

$=7y+77z−21$

Add: $4y^2+44yz−12y+7y+77z−21$

Combine like terms: $−12y+7y=−5y$

Therefore, $4y^2+44yz−5y+77z−21$

Q5. Expand

(i) $(a - b)(a + b)$

Ans:

$=a(a+b)-b(a+b)$

$=a^2+ab-ab-b^2$

The middle terms cancel: a^2-b^2

So, $(a-b)(a+b)=a^2-b^2$

(ii) $(a - b)(a^2 + ab + b^2)$ and

Ans:

Multiply a by every term:

$a(a^2+ab+b^2)=a^3+a^2b+ab^2$

Multiply $-b$ by every term:

$-b(a^2+ab+b^2)=-a^2b-ab^2-b^3$

Add: $a^3+a^2b+ab^2-a^2b-ab^2-b^3$

The middle terms cancel: a^3-b^3

Therefore, $(a-b)(a^2+ab+b^2)=a^3-b^3$

(iii) $(a - b)(a^3 + a^2b + ab^2 + b^3)$

Ans:

Multiply by a:

$a(a^3+a^2b+ab^2+b^3)=a^4+a^3b+a^2b^2+ab^3$

Multiply by $-b$:

$-b(a^3+a^2b+ab^2+b^3)=-a^3b-a^2b^2-ab^3-b^4$

Add: $a^4+a^3b+a^2b^2+ab^3-a^3b-a^2b^2-ab^3-b^4$

All the middle terms cancel: a^4-b^4

Do you see a pattern? What would be the next identity in the pattern that you see? Can you check it by expanding?

Yes! Look at the results:

First: $(a-b)(a+b)=a^2-b^2$

Second: $(a-b)(a^2+ab+b^2)=a^3-b^3$

Third: $(a-b)(a^3+a^2b+ab^2+b^3)=a^4-b^4$

The powers on the right increase by 1 each time.

Therefore, the next identity should be:

$(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4)=a^5-b^5$

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Q1. Which is greater: $(a - b)^2$ or $(b - a)^2$?
Answer:

They are equal.

Since: $b - a = -(a - b)$
Therefore: $(b - a)^2 = [-(a - b)]^2 = (a - b)^2$

$(a - b)^2 = (b - a)^2$

Q2. Express 100 as the difference of two squares.

Using Identity 1C: $a^2 - b^2 = (a + b)(a - b)$

A convenient example is:
$26^2 - 24^2$
$= (26 - 24)(26 + 24)$
$= 2 \times 50$
$= 100$

Therefore: $100 = 26^2 - 24^2$

Q3. Find the following using identities.

(a) 406^2

$406 = 400 + 6$
Therefore:
$406^2 = (400 + 6)^2$
$= 400^2 + 2(400)(6) + 6^2$
$= 160000 + 4800 + 36$
$406^2 = 164836$

(b) 72^2

$72 = 70 + 2$
$72^2 = (70 + 2)^2$
$= 4900 + 280 + 4$
$72^2 = 5184$

(c) 145^2

$145 = 150 - 5$
$145^2 = (150 - 5)^2$
$= 150^2 - 2(150)(5) + 5^2$
$= 22500 - 1500 + 25$
$145^2 = 21025$

(d) 1097^2

$1097 = 1100 - 3$
$1097^2 = (1100 - 3)^2$
$= 1100^2 - 2(1100)(3) + 3^2$
$= 1210000 - 6600 + 9$
$1097^2 = 1,203,409$

(e) 124^2

$124 = 120 + 4$
$124^2 = (120 + 4)^2$
$= 14400 + 960 + 16$
$124^2 = 15376$

Q4. Do Patterns 1 and 2 hold for counting numbers, negative integers and fractions?
Answer:
Yes.

Pattern 1: $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$

Pattern 2: $a^2 - b^2 = (a + b)(a - b)$

Both are algebraic identities. They are obtained by distributive multiplication and therefore remain true for integers, negative integers and fractions.

For example, let: $a = -2$, $b = 1$

Pattern 2:
Left side: $(-2)^2 - 1^2 = 4 - 1 = 3$
Right side: $(-2 + 1)(-2 - 1)$
$= (-1)(-3)$
$= 3$

Both sides are equal.
The same identities also hold for fractions because the algebraic expansion remains valid.

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Q1. Compute using the suggested identities.

(i) 46^2

Use: $(a + b)^2 = a^2 + 2ab + b^2$
Take: $46 = 40 + 6$
Therefore:
$46^2 = (40 + 6)^2$
$= 1600 + 480 + 36$
$= 2116$

(ii) 397×403

Take: $397 = 400 - 3$
$403 = 400 + 3$
Therefore:
397×403
$= (400 - 3)(400 + 3)$

Using Identity 1C:
$= 400^2 - 3^2$
$= 160000 - 9$
$= 159991$

(iii) 91^2

Take: $91 = 90 + 1$
$91^2 = (90 + 1)^2$
$= 8100 + 180 + 1$
$= 8281$

(iv) 43×45

Take:
$43 = 44 - 1$
$45 = 44 + 1$

Therefore:
43×45
$= (44 - 1)(44 + 1)$
$= 44^2 - 1^2$
$= 1936 - 1$
$= 1935$

Q2. Use either a suitable identity or the distributive property to find each of the following products.

(i) $(p - 1)(p + 11)$

$= p^2 + 11p - p - 11$
$= p^2 + 10p - 11$

(ii) $(3a - 9b)(3a + 9b)$

Using $(x - y)(x + y) = x^2 - y^2$:
$= (3a)^2 - (9b)^2$
$= 9a^2 - 81b^2$

(iii) $-(2y + 5)(3y + 4)$

First expand:
$(2y + 5)(3y + 4)$
$= 6y^2 + 8y + 15y + 20$
$= 6y^2 + 23y + 20$
Therefore:
$-(2y + 5)(3y + 4) = -6y^2 - 23y - 20$

(iv) $(6x + 5y)^2$

Using: $(a + b)^2 = a^2 + 2ab + b^2$
$= (6x)^2 + 2(6x)(5y) + (5y)^2$
$= 36x^2 + 60xy + 25y^2$

(v) $(2x - \frac{1}{2})^2$

Using:$(a - b)^2 = a^2 - 2ab + b^2$
$= (2x)^2 - 2(2x)(\frac{1}{2}) + (\frac{1}{2})^2$
$= 4x^2 - 2x + \frac{1}{4}$

(vi) $(7p)(3r)(p + 2)$

First: $7p \times 3r = 21pr$

Therefore:
$21pr(p + 2)$
$= 21p^2r + 42pr$
$= 21p^2r + 42pr$

Q3. For each statement identify the appropriate algebraic expression(s).

(i) Two more than a square number

Let the square number be s^2.
Two more than it is: s^2+2
From the given options, the correct expression is s^2+2.

(ii) The sum of the squares of two consecutive numbers

Let the first number be m.
The next consecutive number is:$m+1$

Their squares are: m^2 and $(m+1)^2$
Therefore, their sum is: $m^2+(m+1)^2$

Another equivalent expression from the options is obtained by taking the consecutive numbers as $m-1$ and m:
$m^2+(m-1)^2$

So the appropriate expressions are: $m^2+(m+1)^2$ and $m^2+(m-1)^2$

Q4. Consider any 2 by 2 square of numbers in a calendar, as shown in the figure.

Find products of numbers lying along each diagonal — 4 × 12 = 48, 5 × 11 = 55. Do this for the other 2 by 2 squares. What do you observe about the diagonal products? Explain why this happens.

Ans:

The four numbers in any 2×2 calendar block can be written as:

a	a+7
a+1	a+8

This is because dates in the next row are exactly 7 greater.
The two diagonal products are:

First diagonal : $a(a+8)=a^2+8a$
Second diagonal: $(a+1)(a+7)$
$=a^2+7a+a+7=a^2+8a+7$

Therefore,$(a+1)(a+7)-a(a+8)=7$
So, The diagonal products always differ by 7.
For the highlighted square:
$4\times12=48$ and$5\times11=55$
Difference: $55-48=7$
Thus, the second diagonal product is always 7 greater than the first diagonal product.

Q5. Verify which of the following statements are true.

(i)

(k+1)(k+2)−(k+3)

Expand:
$=k^2+3k+2-k-3$
$=k^2+2k-1$
This is not always 2.

For example, if k=1:
(2)(3)−4=6−4=2

But if k=2:
(3)(4)−5=12−5=7

Therefore: False

(ii)

(2q+1)(2q−3)

Expand:
$=4q^2-6q+2q-3$
$=4q^2-4q-3$

The first two terms are divisible by 4, but:
$4q^2-4q-3$
leaves remainder 1 when divided by 4.

For example, q=1:
(3)(−1)=−3
which is not divisible by 4.

Therefore: False

(iii)
Square of an even number

Any even number can be written as: $2n$
Its square: $(2n)^2$
$=4n^2$

Therefore, its square is always a multiple of 4.

Square of an odd number

Any odd number can be written as: $2n+1$

Its square: $(2n+1)^2$
$=4n^2+4n+1$
$=4n(n+1)+1$

Since $n(n+1)$ is even, $4n(n+1)$ is a multiple of 8.
Therefore, an odd square is:
1 more than a multiple of 8
Hence: True

(iv)
$(6n+2)^2-(4n+3)^2$

Using: $A^2-B^2=(A-B)(A+B)$
we get:
$=[(6n+2)-(4n+3)][(6n+2)+(4n+3)]$
$=(2n-1)(10n+5)$
$=5(2n-1)(2n+1)$

Now: $(2n-1)(2n+1)=4n^2-1$
Therefore: $=5(4n^2-1)=20n^2-5$

this is not necessarily 5 less than a square number.

Check $n=1$:
$(8)^2-(7)^2$
$=64-49=15$

15 is 5 less than 20, but 20 is not a square.

Therefore: False

Q6. A number leaves a remainder of 3 when divided by 7, and another number leaves a remainder of 5 when divided by 7. What is the remainder when their sum, difference, and product are divided by 7?

Ans:

One number leaves remainder 3 when divided by 7.
Let it be: $7a+3$

The other leaves remainder 5.
Let it be: $7b+5$

(i) Their sum

$(7a+3)+(7b+5)$
$=7a+7b+8$
$=7(a+b+1)+1$
Therefore, the remainder is: 1

(ii) Their difference

Taking the first number minus the second:
$(7a+3)-(7b+5)$
$=7a-7b-2$
$=7(a-b-1)+5$
Therefore, the remainder is: 5

(iii) Their product

$(7a+3)(7b+5)$
The remainder depends only on: $3\times5=15$
Now: $15=7\times2+1$
Therefore, the remainder is: 1
Final Answer: Sum=1,Difference=5,Product=1

Q7. Choose three consecutive numbers, square the middle one, and subtract the product of the other two. Repeat the same with other sets of numbers. What pattern do you notice? How do we write this as an algebraic equation? Expand both sides of the equation to check that it is a true identity.

Ans:

Let them be:

$n-1, n, n+1$

The middle number is n.

Square the middle number: n^2
Product of the other two: $(n-1)(n+1)$
Using difference of squares: $(n-1)(n+1)=n^2-1$

Now subtract:

$n^2-(n^2-1)=n^2-n^2+1=1$

Therefore, no matter which three consecutive numbers we choose, the result is always 1.

Example

Take: 4, 5, 6
Square middle number: $5^2=25$
Product of the other two: $4\times6=24$
Difference: $25-24=1$

Algebraic identity: $n^2-(n-1)(n+1)=1$

Q8. What is the algebraic expression describing the following steps—add any two numbers. Multiply this by half of the sum of the two numbers? Prove that this result will be half of the square of the sum of the two numbers.

Ans:

Let the two numbers be: a and b
Their sum: $a+b$
Half of their sum: $(a+b)/2$

Therefore, the required algebraic expression is:

$(a+b)(a+b)/2$

Simplify: $=(a+b)^2/2$

Now expand:
$(a+b)^2/2 = (a^2+2ab+b^2)/2$

Thus, the result is exactly:
half of the square of the sum

9. Which is larger? Find out without fully computing the product.
(i) 14 × 26 or 16 × 24

Ans:

$14\times26=(20-6)(20+6)$

Using: $(a-b)(a+b)=a^2-b^2=20^2-6^2$
Similarly: $16\times24=(20-4)(20+4) =20^2-4^2$
Since: $4^2<6^2$
we have: $20^2-4^2 > 20^2-6^2$
Therefore:16×24 is larger

(ii) 25 × 75 or 26 × 74

Ans:

Write both around 50:

$25\times75 = (50-25)(50+25)$
$=50^2-25^2$

$26\times74=(50-24)(50+24)$
$=50^2-24^2$

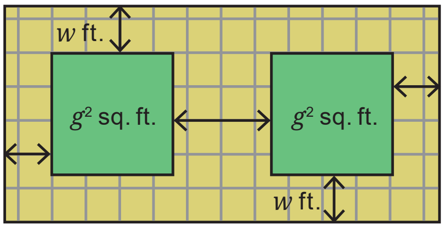
Since: $24^2<25^2$

we get: $50^2-24^2 > 50^2-25^2$

Therefore: 26×74 is larger

10. A tiny park is coming up in Dhauli. The plan is shown in the figure. The two square plots, each of area g^2 sq. ft., will have a green cover. All the remaining area is a walking path w ft. wide that needs to be tiled. Write an expression for the area that needs to be tiled.

Ans:



Total length
$w+g+w+g+w$
So: $2g+3w$

Total width
$w+g+w$
So: $g+2w$

Therefore, total park area is:
$(2g+3w)(g+2w)$

The two green squares have total area:
$2g^2$

Therefore, tiled walking-path area is:
$(2g+3w)(g+2w)-2g^2$

Expand:
$=2g^2+4gw+3gw+6w^2-2g^2$
$=7gw+6w^2$

Hence: Area to be tiled$=7gw+6w^2$ sq. ft.