4 Quadrilaterals

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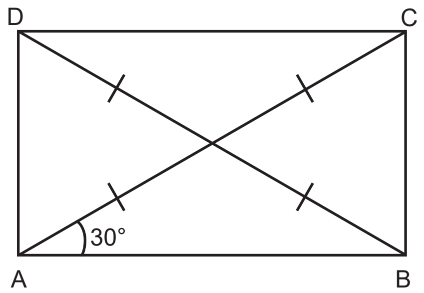
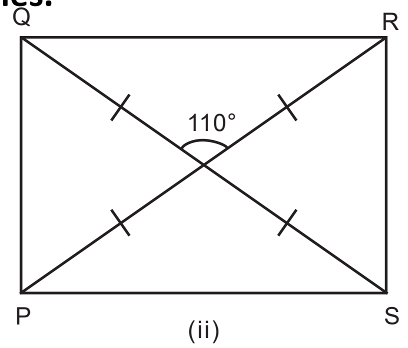
Q1. Find all other angles inside the given rectangles.



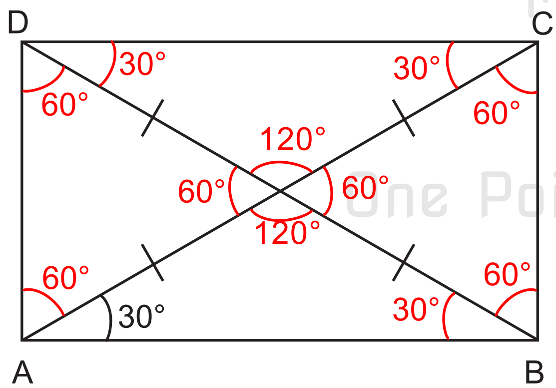
(i)



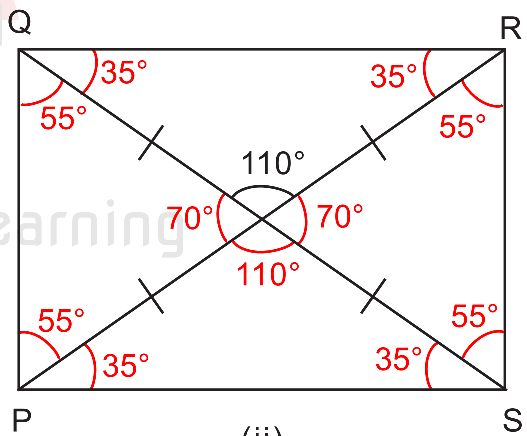
(ii)

Solution:
From the properties of rectangle

- All the corners of a rectangle are 90°
- Opposite sides are equal and parallel
- Diagonals are equal and bisect each other
- Vertically opposite angles
- Interior angles of a triangle



(i)

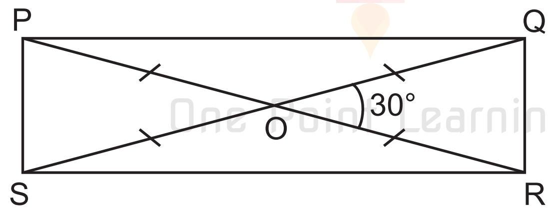


(ii)

Q2. Draw a quadrilateral whose diagonals have equal lengths of 8 cm that bisect each other, and intersect at an angle of:

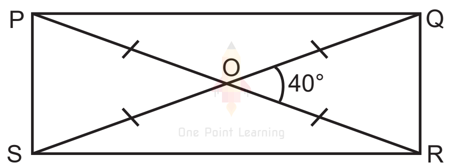
(i) 30°

- Draw a line PR equal to 8 cm.
- Take a midpoint O on PR.
- Using a protractor, with point on O and base on PR mark point at 30°.
- Draw a line QS of 8 cm at 30° and whose midpoint lies at O.
- Join PQ, QR, RS, and SP.
- PQRS is the required quadrilateral

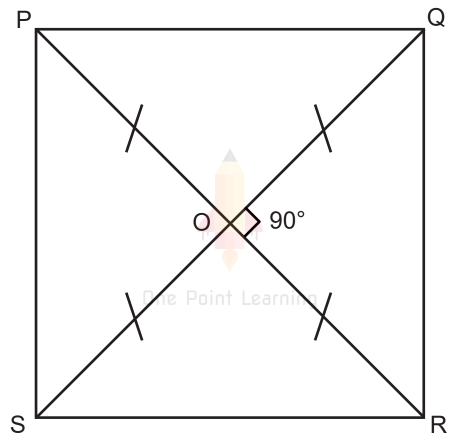


Using the same process as above draw all quadrilaterals.
In all cases except 90°, you get a rectangle. At 90°, you get a square.

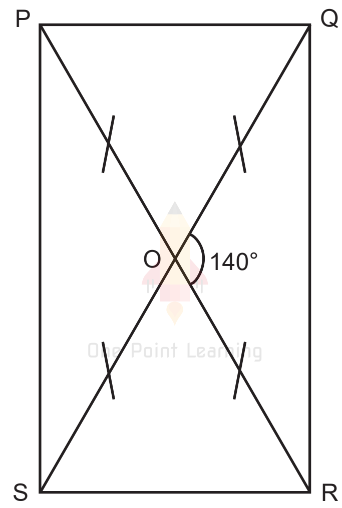
(ii) 40° → Rectangle



(iii) 90° → Square



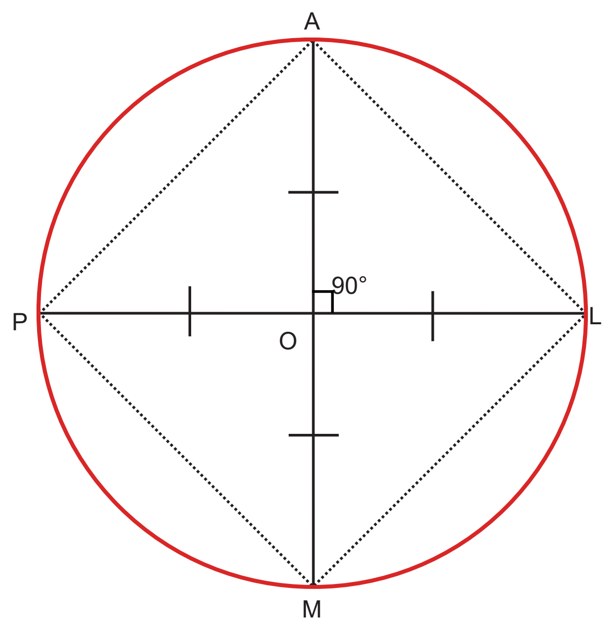
(iv) 140° → Rectangle



Q3. Consider a circle with centre O. Line segments PL and AM are two perpendicular diameters of the circle. What is the figure APML?

Solution:

- As per the properties of circle diameter is a line segment passing through the centre and connecting two points on a circle.
- Also all the points on a circle are equidistance from the centre, thus all diameters are equal.
- Now, since diagonals are equal and bisect each other at 90°, **APML is a square**.



Q4. We have seen how to get 90° using paper folding. Now, suppose we do not have any paper but two sticks of equal length, and a thread. How do we make an exact 90° using these?

Solution: Place the two equal sticks as diagonals crossing at their midpoints and tie them with a thread. Since equal diagonals that bisect each other intersect at right angles in a square, this arrangement gives an exact 90° angle.

Q5. We saw that one of the properties of a rectangle is that its opposite sides are parallel. Can this be chosen as a definition of a rectangle?
In other words, is every quadrilateral that has opposite sides parallel and equal, a rectangle?

Solution: No. That defines a parallelogram. A rectangle is a special parallelogram with all angles = 90°.

A quadrilateral with only opposite sides parallel and equal is not necessarily a rectangle.

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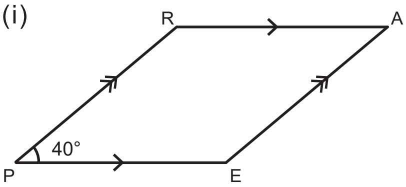
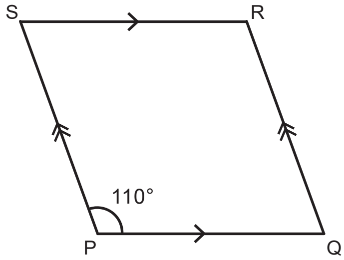
Q1. Find the remaining angles in the following quadrilaterals.

Solution:

(i)



(ii)

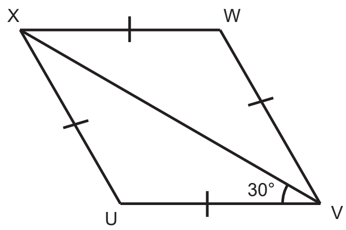


In a parallelogram opposite angles are equal
Also sum of all angles is 360°
Thus, sum of adjecent angle is 180°

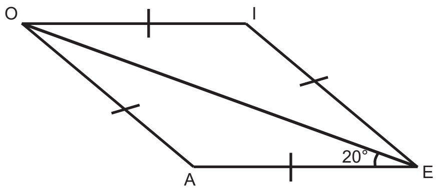
∠RPE = ∠EAR = 40°
∠PEA = ∠PRA = 140°

∠SPQ = ∠SRQ = 110°
∠PSR = ∠PQR = 70°

(iii)



(iv)



Here the given quadrilaterals are Rohmbus, as all sides are equal
Using properties of Rohmbus

∠UVX = ∠UXV = 30°
∠UXW = ∠UVW = 60°
∠XUV = ∠XWV = 120°

∠AEO = ∠AOE = 20°
∠AEI = ∠AOI = 40°
∠OAE = ∠OIE = 140°

Q2. Using the diagonal properties, construct a parallelogram whose diagonals are of lengths 7 cm and 5 cm, and intersect at an angle of 140°.

Solution:

Given:

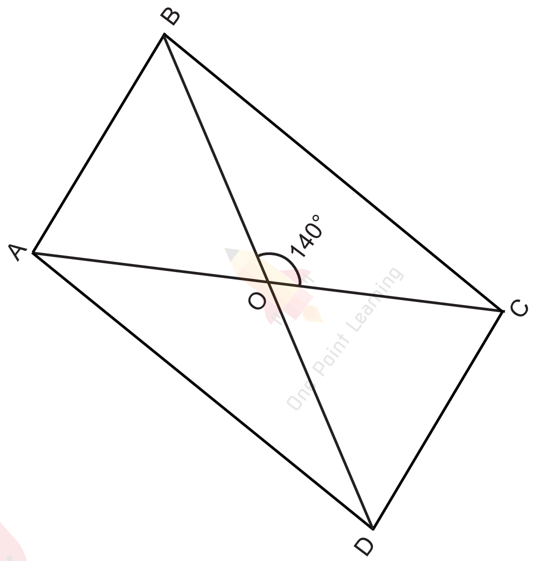
Diagonal AC = 7 cm

Diagonal BD = 5 cm

∠AOB = 140°, where O is the point of intersection of the diagonals.

Property Used:

The diagonals of a parallelogram bisect each other.



Steps:

Draw a line segment AC = 7 cm.

Mark its midpoint O.

OA = OC = 3.5 cm.

At O, construct an angle ∠AOB = 140°.

On ray OB, mark OB = 2.5 cm.

On the opposite ray OD, mark OD = 2.5 cm.

Thus, BD = 5 cm.

Join A to B, B to C, C to D, and D to A.

Result: ABCD is the required parallelogram.

Q3. Using the diagonal properties, construct a rhombus whose diagonals are of lengths 4 cm and 5 cm.

Solution:

Given:

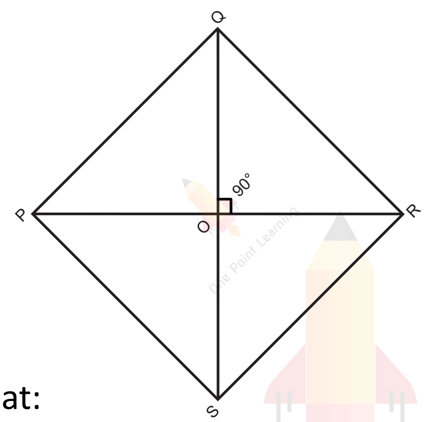
One diagonal = 5 cm

Other diagonal = 4 cm

Properties Used:

The diagonals of a rhombus bisect each other.

The diagonals of a rhombus intersect at 90°.



Steps:

Draw a line segment PR = 5 cm.

Mark its midpoint O.

OP = OR = 2.5 cm.

Through O, draw a line perpendicular to PR.

On the perpendicular line, mark points Q and S such that:

OQ = OS = 2 cm

Therefore, QS = 4 cm.

Join P to Q, Q to R, R to S, and S to P.

Result: PQRS is the required rhombus.

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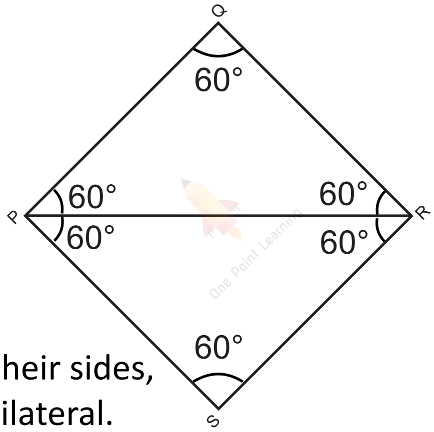
Q1. Find all the sides and the angles of the quadrilateral obtained by joining two equilateral triangles with sides 4 cm.

Solution:



An equilateral triangle has:

All sides = 4 cm

Each angle = 60°

When two equilateral triangles are joined along one of their sides, the common side becomes an internal side of the quadrilateral.

Therefore, the four outer sides of the quadrilateral are: 4 cm, 4 cm, 4 cm, 4 cm

So, all four sides are equal.

At the two vertices where the triangles meet, the angles remain: 60°

At the other two vertices, two angles of 60° combine to form: 60° + 60° = 120°

Therefore, the four angles are: 60°, 120°, 60°, 120°

Hence, the quadrilateral is a rhombus.

Q2. Construct a kite whose diagonals are of lengths 6 cm and 8 cm.

Solution:

A kite has two pairs of adjacent equal sides. Its symmetry diagonal is perpendicular to and bisects the other diagonal.

Construction Steps:

Draw a line segment AB = 6 cm.

This will be one diagonal of the kite.

Take a midpoint O on AB.

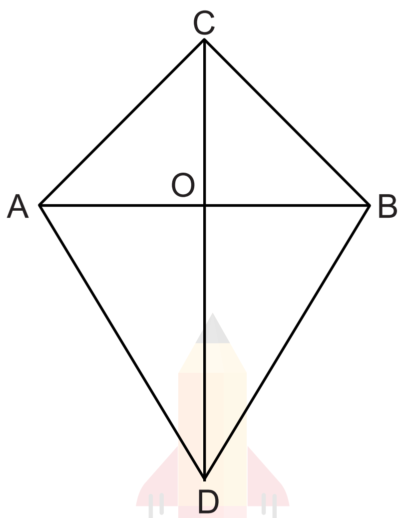
Through O, construct a line perpendicular to AB.

On this perpendicular line, mark points C and D such that:

OC = 3 cm and OD = 5 cm

Therefore, CD=3+5=8 cm

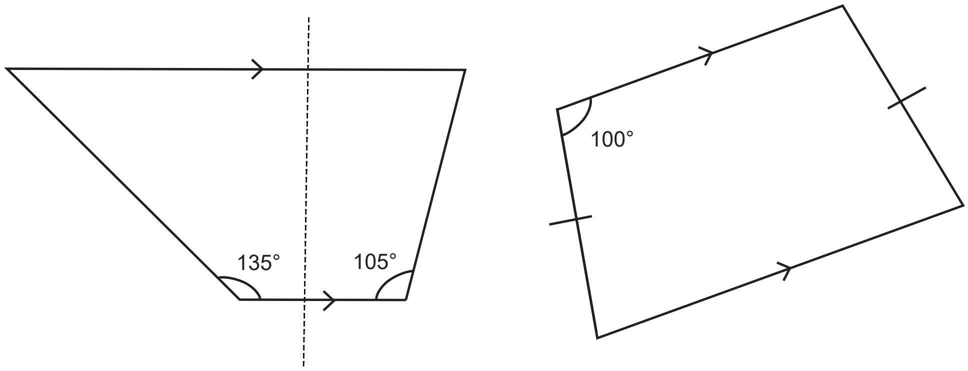
Join: A–C,C–B,B–D,D–A



The resulting quadrilateral ACBD is the required kite.

Q3. Find the remaining angles in the following trapeziums

Solution:



(i) First trapezium

Given angles: 135° and 105°
Since the two parallel sides are cut by transversals, the interior angles on the same side add up to 180°
For the first pair: Missing angle = 180° − 135° = 45°
For the second pair: Missing angle = 180° − 105° = 75°

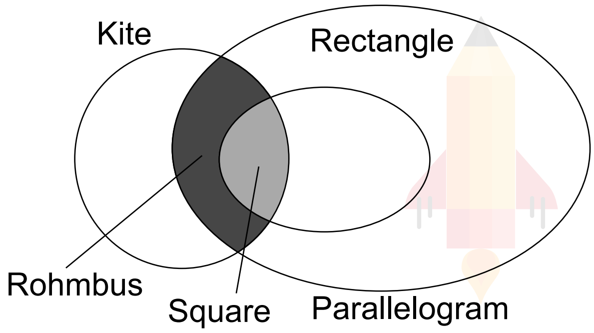
(ii) Second trapezium

The given angle is 100°, and the non-parallel sides are equal. Therefore, it is an isosceles trapezium.
In an isosceles trapezium, the angles adjacent to the same base are equal.
Therefore, the other angle on the same base is: 100°.
Angles on the same side of a transversal add up to 180°.
Hence, 180° - 100° = 80°

Therefore, the remaining angles are: 100°, 80°, 80°

Q4. Draw a Venn diagram showing the set of parallelograms, kites, rhombuses, rectangles, and squares. Then, answer the following questions
(i) What is the quadrilateral that is both a kite and a parallelogram?
(ii) Can there be a quadrilateral that is both a kite and a rectangle?
(iii) Is every kite a rhombus? If not, what is the correct relationship between these two types of quadrilaterals?

Solution:
The relationships are:



- Every rectangle is a parallelogram.
- Every rhombus is a parallelogram.
- Every rhombus is also a kite.
- Every square is a rectangle.
- Every square is a rhombus.

Therefore, every square is also a kite and a parallelogram.

(i) What is the quadrilateral that is both a kite and a parallelogram?

A rhombus.

(ii) Can there be a quadrilateral that is both a kite and a rectangle?

Yes. A square satisfies both definitions.
A square has:
Two pairs of adjacent equal sides → kite
Four right angles → rectangle

(iii) Is every kite a rhombus?

No.
A kite has two pairs of adjacent equal sides, whereas a rhombus has all four sides equal.
However: Every rhombus is a kite, but every kite is not a rhombus.

Q5. If PAIR and RODS are two rectangles, find ∠IOD.

Solution:

From the figure: ∠IRO=30°
Since PAIR is a rectangle, ∠RIO=90°

Consider △RIO. Using the angle-sum property of a triangle:
∠IRO + ∠RIO + ∠IOR = 180°

Substitute the known values:
30° + 90° + ∠IOR = 180°
120° + ∠IOR = 180°
Therefore, ∠IOR = 60°

Now, RODS is also a rectangle, so:∠ROD=90°
But:∠ROD = ∠ROI + ∠IOD
Therefore,90°= 60° + ∠IOD
∠IOD=90°−60°
∠IOD=30°

Q6. Construct a square with diagonal 6 cm without using a protractor.

Solution:

Draw a line segment: AC=6 cm
Construct the perpendicular bisector of AC.
Let the perpendicular bisector meet AC at O.
Therefore: AO = OC = 3 cm
With O as centre and radius 3 cm, mark points B and D on the perpendicular bisector such that: OB = OD = 3 cm
Join: A–B–C–D–A
The resulting quadrilateral ABCD is the required square.

Q7. CASE is a square. The points U, V, W and X are the midpoints of the sides of the square. What type of quadrilateral is UVWX? Find this by using geometric reasoning, as well as by construction and measurement. Find other ways of constructing a square within a square such that the vertices of the inner square lie on the sides of the outer square, as shown in Figure (b).

Solution:

Since U,V,W,X are the midpoints of the four sides of square CASE:

The segments joining consecutive midpoints are equal.
The diagonals of the inner quadrilateral are equal.
Its diagonals intersect at right angles.

Therefore, UV=VW=WX=XU
and all its angles are 90°.

Hence: UVWX is a square

Q8. If a quadrilateral has four equal sides and one angle of 90°, will it be a square? Find the answer using geometric reasoning as well as by construction and measurement.

Solution:

A quadrilateral with all four sides equal is a rhombus.
Suppose one angle is: 90°
Since a rhombus is a parallelogram, its adjacent angles are supplementary.
Therefore, the adjacent angle is: 180°−90°=90°

Similarly, the opposite angles are equal.
Hence, all four angles are: 90°

So the quadrilateral has:
all four sides equal;
all four angles 90°.
Therefore, it is a square.

Q9. What type of a quadrilateral is one in which the opposite sides are equal? Justify your answer. Hint: Draw a diagonal and check for congruent triangles.

Solution: It is a parallelogram.

Let ABCD be a quadrilateral such that: AB=CD and AD=BC
Draw diagonal AC.
Now consider triangles: $\triangle$ABC and $\triangle$CDA
We have:
AB=CD
BC=AD and
AC=AC, because AC is common.

Therefore, $\triangle ABC \cong \triangle CDA$ by SSS congruence.
Hence, corresponding angles are equal: $\angle BAC = \angle DCA$

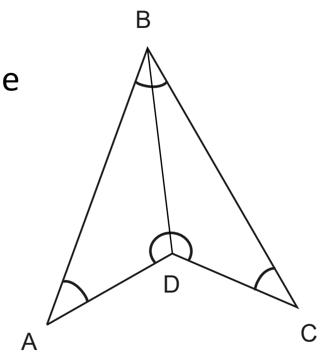
These are alternate interior angles, so: $AB \parallel CD$
Similarly, $AD \parallel BC$
Thus, both pairs of opposite sides are parallel.

Therefore: ABCD is a parallelogram

Q10. Will the sum of the angles in a quadrilateral such as the following one also be 360°? Find the answer using geometric reasoning as well as by constructing this figure and measuring.

Solution:

Yes. The quadrilateral shown in the question is a concave quadrilateral.



Draw a suitable diagonal from one vertex to another so that the figure is divided into two triangles.
The sum of the angles of each triangle is: 180°
Therefore, the total is: 180° + 180° = 360°
Hence, Sum of the angles = 360°

Also by measuring the angles in figure we get
$\angle A=40° + \angle B=50° + \angle C=40° + \angle D=230° = 360°$
Sum of the angles in a quadrilateral is 360°

Q11. State whether the following statements are true or false. Justify your answers.

(i) A quadrilateral whose diagonals are equal and bisect each other must be a square.
(ii) A quadrilateral having three right angles must be a rectangle.
(iii) A quadrilateral whose diagonals bisect each other must be a parallelogram.
(iv) A quadrilateral whose diagonals are perpendicular to each other must be a rhombus.
(v) A quadrilateral in which the opposite angles are equal must be a parallelogram.
(vi) A quadrilateral in which all the angles are equal is a rectangle.
(vii) Isosceles trapeziums are parallelograms.

Solution:

1. False
2. True
3. True
4. False
5. True
6. True
7. False