5 PRIME TIME

5.1: Common Multiples and Common Factors

Idli-Vada Game Questions:

Q1. At what number is 'idli-vada' said for the 10th time?

Ans: 10th 'idli-vada': 150 (LCM of 3 and 5 = 15 → 10 × 15 = 150)

Q2. If the game is played for the numbers 1 to 90, find out:

(a) How many times would the children say 'idli' (including the times they say 'idli-vada')?

Ans: 'idli' (multiples of 3): 30 times

(b) How many times would the children say 'vada' (including the times they say 'idli-vada')?

Ans: 'vada' (multiples of 5): 18 times

(c) How many times would the children say 'idli-vada'?

Ans: 'idli-vada' (common multiples of 3 and 5): 6 times

Q3. What if the game was played till 900? How would your answers change?

(a) idli = 300, (b) vada = 180, (c) idli-vada = 60

Q4. Is this figure 5.1 somehow related to the 'idli-vada' game?

Ans: Yes, the figure is related to the game. The venn diagram shows how many time Idli, Vada and Idli-vada is said.

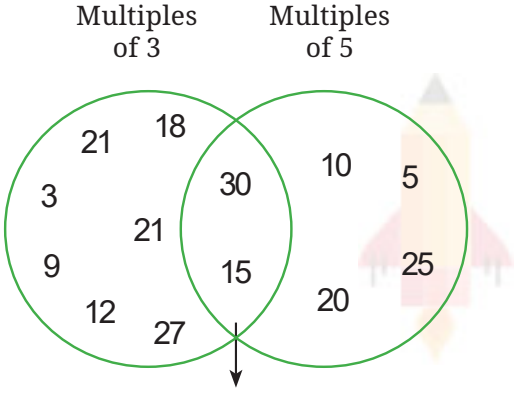


Fig. 5.1

Other pairs:

a. 2 & 5 → LCM: 10

b. 3 & 7 → LCM: 21

c. 4 & 6 → LCM: 12

Figure it Out

1. Find all multiples of 40 that lie between 310 and 410.

Ans: Multiples of 40 between 310 and 410: 320, 360, 400

2. Who am I?

a. I am a number less than 40. One of my factors is 7. The sum of my digits is 8.

Ans: 32 (Factors: 1, 5, 7, 35; 3+5=8)

b. I am a number less than 100. Two of my factors are 3 and 5.

One of my digits is 1 more than the other.

Ans: 45 (Factors include 3 & 5; digits: 4 and 5)

3. A number for which the sum of all its factors is equal to twice the number is called a perfect number. The number 28 is a perfect number. Its factors are 1, 2, 4, 7, 14 and 28. Their sum is 56 which is twice 28. Find a perfect number between 1 and 10.

Ans: Perfect number between 1 and 10: 6 (1 + 2 + 3 + 6 = 12 = 2×6)

4. Find the common factors of:

a. 20 and 28 = 1, 2, 4

b. 35 and 50 = 1, 5

c. 4, 8 and 12 = 1, 2, 4

d. 5, 15 and 25 = 1, 5

5. Find any three numbers that are multiples of 25 but not multiples of 50.

Ans: three numbers are 25, 75, 125

6. Anshu and his friends play the 'idli-vada' game with two numbers, which are both smaller than 10. The first time anybody says 'idlivada' is after the number 50. What could the two numbers be which are assigned 'idli' and 'vada'?

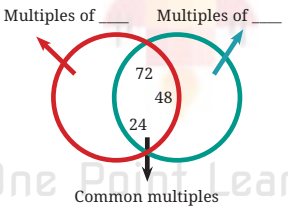
Ans: possible numbers → 6 and 9 (LCM = 54)

7. In the treasure hunting game, Grumpy has kept treasures on 28 and 70. What jump sizes will land on both the numbers?

Ans: We need to find common factors of 28 and 70. The numbers are 2 and 7.

8. In the diagram below, Guna has erased all the numbers except the common multiples. Find out what those numbers could be and fill in the missing numbers in the empty regions.

Ans: Multiples of 24 and 72 (LCM = 72), possible: 24, 48, 72



9. Find the smallest number that is a multiple of all the numbers from 1 to 10, except for 7.

Ans: Smallest multiple of 1–10 (except 7): LCM = 2520 (excluding 7 → LCM = 360)

10. Find the smallest number that is a multiple of all the numbers from 1 to 10.

Ans: Smallest multiple of 1–10: LCM = 2520

5.2: Prime Numbers

1. We see that 2 is a prime and also an even number. Is there any other even prime?

Ans: No, only 2 is such a number.

2. Look at the list of primes till 100. What is the smallest difference between two successive primes? What is the largest difference?

Ans: Smallest difference between primes till 100 is 2 (3 & 5) and largest difference between primes till 100 = 8 (89 & 97)

3. Are there an equal number of primes occurring in every row in the table on the previous page? Which decades have the least number of primes? Which have the most number of primes?

Ans: No, number of primes are different. Decade 91 to 100 have only 1 prime, while 1 to 10 and 11 to 20 have 4 primes each.

4. Which of the following numbers are prime: 23, 51, 37, 26?

Ans: 23 → Prime, 51 → Not (3 × 17), 37 → Prime, 26 → Not (2 × 13)

5. Write three pairs of prime numbers less than 20 whose sum is a multiple of 5.

Ans: (2, 3), (7, 13), (3, 7)

6. The numbers 13 and 31 are prime numbers. Both these numbers have same digits 1 and 3. Find such pairs of prime numbers up to 100.

Ans: (13, 31), (17, 71), (37, 73), (79, 97)

7. Find seven consecutive composite numbers between 1 and 100.

Ans: Seven consecutive composites: 90, 91, 92, 93, 94, 95, 96

8. Twin primes are pairs of primes having a difference of 2. For example, 3 and 5 are twin primes. So are 17 and 19. Find the other twin primes between 1 and 100.

Ans: (11,13), (17,19), (29,31), (41,43), (59,61), (71,73)

9. Identify whether each statement is true or false. Explain.

a. There is no prime number whose units digit is 4. **- True**

b. A product of primes can also be prime. **- False**

c. Prime numbers do not have any factors. **- False**

d. All even numbers are composite numbers.**- False**

e. 2 is a prime and so is the next number, 3. For every other prime, the next number is composite. **- True**

10. Which of the following numbers is the product of exactly three distinct prime numbers: 45, 60, 91, 105, 330?

Ans: 105 = 3×5×7

11. How many three-digit prime numbers can you make using each of 2, 4 and 5 once?

Ans: No prime numbers

12. Observe that 3 is a prime number, and 2 × 3 + 1 = 7 is also a prime. Are there other primes for which doubling and adding 1 gives another prime? Find at least five such examples.

Ans:

2×2+1=5

2×3+1=7

2×5+1=11

2×11+1=23

2×23+1=47

5.3 Co-prime numbers for safekeeping treasures

Q. Check if these pairs are safe:

Safe pairs: (no common factors except 1)

a. 15 & 39 → No

b. 4 & 15 → Yes

c. 18 & 29 → Yes

d. 20 & 55 → No

Which of the following pairs of numbers are co-prime?

a. 18 & 35 → Yes

b. 15 & 37 → Yes

c. 30 & 415 → No (5 common)

d. 17 & 69 → Yes

e. 81 & 18 → No

5.4: Prime Factorisation

1. Find the prime factorisations of the following numbers:

- 64 = 2 × 2 × 2 × 2 × 2 × 2
- 104 = 2 × 2 × 2 × 13
- 105 = 3 × 5 × 7
- 243 = 3 x 3 x 3 x 3 x 3
- 320 = 2 × 2 × 2 × 2 × 2 × 2 × 5
- 141 = 3 × 47
- 1728 = 2 × 2 × 2 × 2 × 2 × 2 × 3 x 3 x 3
- 729 = 3 x 3 x 3 x 3 x 3 x 3
- 1024 = 2 × 2 × 2 × 2 × 2 x 2 × 2 × 2 × 2 × 2
- 1331 = 11 x 11 x 11
- 1000 = 2 × 2 × 2 × 5 x 5 x 5

2. The prime factorisation of a number has one 2, two 3s, and one 11. What is the number?

Ans: 2 × 3 × 3 × 11 = 198

3. Find three prime numbers, all less than 30, whose product is 1955.

Ans: 5 × 17 × 23 = 1955

4. Find the prime factorisation of these numbers without multiplying first

a. 56 × 25 = (2 x 2 x 2 x 7) x (5 x 5)

b. 108 × 75 = (2 x 2 x 3 x 3 x 3) x (5 x 5 x 3)

c. 1000 × 81 = (2 x 2 x 2 x 5 x 5 x 5) x (3 x 3 x 3 x 3)

5. What is the smallest number whose prime factorisation has:

a. three different prime numbers? **2 × 3 × 5 = 30**

b. four different prime numbers? **2 × 3 × 5 × 7 = 210**

Using prime factorisation to check if one number is divisible by another

Figure it out:

1. Are the following pairs of numbers co-prime? Guess first and then use prime factorisation to verify your answer.

a. 30 and 45

Prime factors of 30 = 2 × 3 × 5

Prime factors of 45 = 3 × 3 × 5

Common factors = 3 and 5

-- Not co-prime --

b. 57 and 85

Prime factors of 57 = 3 × 19

Prime factors of 85 = 5 × 17

No common prime factor

-- Co-prime --

c. 121 and 1331

Prime factors of 121 = 11 × 11

Prime factors of 1331 = 11 × 11 × 11

Common factor = 11

-- Not co-prime --

d. 343 and 216

Prime factors of 343 = 7 × 7 × 7

Prime factors of 216 = $2^3 \times 3^3$

No common factor

-- Co-prime --

2. Is the first number divisible by the second? Use prime factorisation.

Definition: A number A is divisible by B if all the prime factors of B are present in A (with equal or higher powers).

a. 225 and 27

225 = $3^2 \times 5^2$

27 = 3^3

225 does not have enough 3s → Not divisible

b. 96 and 24

$96 = 2^5 \times 3$

$24 = 2^3 \times 3$

All prime factors of 24 are in 96 → Divisible

c. 343 and 17

$343 = 7^3$

17 is a prime not in 343 → Not divisible

d. 999 and 99

$999 = 3^3 \times 37$

$99 = 3^2 \times 11$

11 is not a factor of 999 → Not divisible

3. The first number has prime factorisation 2 × 3 × 7 and the second number has prime factorisation 3 × 7 × 11. Are they co-prime? Does one of them divide the other?

Ans:

First number: $2 \times 3 \times 7 = 42$

Second number: $3 \times 7 \times 11 = 231$

Common prime factors = 3 and 7

Not co-prime

Does 42 divide 231?

$231 \div 42 = 5.5$ → Not a whole number

No, neither divides the other

4. Guna says, "Any two prime numbers are co-prime?". Is he right?

Yes, Guna is correct.

Prime numbers have only 1 and themselves as factors.

So, any two different prime numbers will not share any common factor except 1.

Hence, they are always co-prime.

Section 5.5: Divisibility Tests

1. 2024 is a leap year (as February has 29 days). Leap years occur in the years that are multiples of 4, except for those years that are evenly divisible by 100 but not 400.

a. From the year you were born till now, which years were leap years?

Ans: 2013 to 2024 - 2016, 2020, 2024 - 3 leap years

b. From the year 2024 till 2099, how many leap years are there?

Ans: Every 4 years → total = 19

2. Find the largest and smallest 4-digit numbers that are divisible by 4 and are also palindromes.

Ans:

Smallest 4-digit palindrome divisible by 4 = 2112

Largest 4-digit palindrome divisible by 4 = 8888

3. Explore and find out if each statement is always true, sometimes true or never true. You can give examples to support your reasoning.

a. Sum of two even numbers gives a multiple of 4.

Ans: always even, may or may not be a multiple of 4

b. Sum of two odd numbers gives a multiple of 4.

Ans: always even, may or may not be a multiple of 4

4. Find the remainders obtained when each of the following numbers are divided by a) 10, b) 5, c) 2.

Ans:

	Divisible : Remainder		
a. 78 →	10: 8,	5: 3,	2: 0
b. 99 →	10: 9,	5: 4,	2: 1
c. 173 →	10: 3,	5: 3,	2: 1
d. 572 →	10: 2,	5: 2,	2: 0
e. 980 →	10: 0,	5: 0,	2: 0
f. 1111 →	10: 1,	5: 1,	2: 1
g. 2345 →	10: 5,	5: 0,	2: 1

5. The teacher asked if 14560 is divisible by all of 2, 4, 5, 8 and 10. Guna checked for divisibility of 14560 by only two of these numbers and then declared that it was also divisible by all of them. What could those two numbers be?

Ans Checked only divisibility by 4 and 5 (LCM = 20) → implies divisibility by 2, 8, 10

6. Which of the following numbers are divisible by all of 2, 4, 5, 8 and 10: 572, 2352, 5600, 6000, 77622160.

Ans:

572 - not divisible

2352 - not divisible

5600 - divisible

6000 - divisible

77622160 - divisible

7. Write two numbers whose product is 10000. The two numbers should not have 0 as the units digit.

Ans: two numbers are 125 × 80