Integers

Exercise 1.1

1. Write down a pair of integers whose:

Answer:
$$-3+(-4)=-7$$

Answer:
$$5-15=-10$$

Answer:
$$6+(-6)=0$$

2.

(a) Write a pair of negative integers whose difference gives 8.

Answer:
$$-2-(-10)=-2+10=8$$

(b) Write a negative integer and a positive integer whose sum is −5.

Answer:
$$-9+4=-5$$

(c) Write a negative integer and a positive integer whose difference is -3.

Answer:
$$-5-(-2)=-5+2=-3$$

3. In a quiz, team A scored – 40, 10, 0 and team B scored 10, 0, – 40 in three successive rounds. Which team scored more? Can we say that we can add integers in any order?

Team A scores:

Total:
$$-40+10+0=-30$$
 Total: $10+0+(-40)=-30$

Answer: Both teams scored –30, so the scores are equal.

Yes! Integer addition is commutative and associative.

Yes, we can add integers in any order.

4. Fill in the blanks:

(i)
$$(-5)+(-8)=(-8)+($$

By commutative property, the blank is -5.

Answer: -5

(ii)
$$-53+$$
___= -53

Adding 0 keeps the number same.

Answer: 0

(iii)
$$17 + = 0$$

Add the additive inverse of 17: –17

Answer: -17

(iv)
$$[13+(-12)]+($$
 $)=13+[(-12)+(-7)]$

Use associative property: the missing number is -7

Answer: -7

$$(\vee) (-4) + [15 + (-3)] = [-4 + 15] + \underline{\hspace{1cm}}$$

Use associative property: the missing number is -3

Answer: -3

Exercise 1.2

1. Find each of the following products:

(a)
$$3 \times (-1) = -3$$

(b)
$$(-1) \times 225 = -225$$

(c)
$$(-21) \times (-30) = 630$$
 (Negative \times Negative $=$ Positive)

(d)
$$(-316) \times (-1) = 316$$

(e)
$$(-15) \times 0 \times (-18) = 0$$
 (Any number $\times 0 = 0$)

(f)
$$(-12) \times (-11) \times (10) = 132 \times 10 = 1320$$

(g)
$$9 \times (-3) \times (-6) = -27 \times (-6) = 162$$

(h)
$$(-18) \times (-5) \times (-4) = 90 \times (-4) = -360$$

(i)
$$(-1) \times (-2) \times (-3) \times 4 = 2 \times (-3) \times 4 = -6 \times 4 = -24$$

(j)
$$(-3) \times (-6) \times (-2) \times (-1) = 18 \times (-2) \times (-1) = -36 \times (-1) = 36$$

2. Verify the following:

(a)
$$18 \times [7 + (-3)] = [18 \times 7] + [18 \times (-3)]$$

LHS:
$$18 \times (4) = 72$$

RHS:
$$126 + (-54) = 72$$

Verified.

(b)
$$(-21) \times [(-4) + (-6)] = [(-21) \times (-4)] + [(-21) \times (-6)]$$

LHS:
$$(-21) \times (-10) = 210$$

RHS:
$$84 + 126 = 210$$

Verified.

3.

(i) For any integer a,
$$(-1) \times a = -a$$

(ii) Determine the integer whose product with (-1) is:

(a)
$$-22 \rightarrow 22$$

(b)
$$37 \rightarrow -37$$

(c)
$$0 \rightarrow 0$$

4. Starting from $(-1) \times 5$, write various products showing some pattern to show $(-1) \times (-1) = 1$.

Let's look at the pattern starting from:

$$(-1) \times 5 = -5$$

$$(-1) \times 4 = -4$$

$$(-1) \times 3 = -3$$

$$(-1) \times 2 = -2$$

$$(-1) \times 1 = -1$$

$$(-1) \times 0 = 0$$

$$(-1) \times (-1) = ?$$

From above, each time the right-hand number decreases by 1, the product increases by 1:

So, next is +1

$$\therefore (-1) \times (-1) = 1$$

Exercise 1.3

1. Evaluate each of the following:

(a)
$$(-30) \div 10 = -3$$

(b)
$$50 \div (-5) = -10$$

(c)
$$(-36) \div (-9) = 4$$

(d)
$$(-49) \div (49) = -1$$

(e)
$$13 \div [(-2) + 1] = 13 \div (-1) = -13$$

(f)
$$0 \div (-12) = 0$$

(g)
$$(-31) \div [(-30) + (-1)] = (-31) \div (-31) = 1$$

(h)
$$[(-36) \div 12] \div 3 = (-3) \div 3 = -1$$

(i)
$$[(-6) + 5] \div [(-2) + 1] = (-1) \div (-1) = 1$$

2. Verify that $a \div (b + c) \neq (a \div b) + (a \div c)$

(a)
$$a = 12$$
, $b = -4$, $c = 2$

LHS:
$$12 \div (-4 + 2) = 12 \div (-2) = -6$$

RHS:
$$(12 \div -4) + (12 \div 2) = (-3) + 6 = 3$$

Verified: LHS ≠ RHS

(b)
$$a = -10$$
, $b = 1$, $c = 1$

LHS:
$$-10 \div (1 + 1) = -10 \div 2 = -5$$

RHS:
$$(-10 \div 1) + (-10 \div 1) = -10 + (-10) = -20$$

Verified: LHS ≠ RHS

3. Fill in the blanks:

(a)
$$369 \div 1 = 369$$

(b)
$$(-75) \div 75 = -1$$

(c)
$$(-206) \div -206 = 1$$

(d)
$$-87 \div -1 = 87$$

(e)
$$-87 \div 1 = -87$$

(f)
$$-48 \div 48 = -1$$

(g)
$$20 \div -10 = -2$$

(h)
$$-12 \div 4 = -3$$

4. Five pairs of integers (a, b) such that $a \div b = -3$:

$$(6, -2) \rightarrow 6 \div (-2) = (-3)$$

$$(-9, 3) \rightarrow -9 \div 3 = -3$$

$$(12, -4) \rightarrow 12 \div (-4) = -3$$

$$(-15, 5) \rightarrow -15 \div 5 = -3$$

$$(21, -7) \rightarrow 21 \div (-7) = -3$$

5. The temperature at 12 noon was 10°C above zero. If it decreases at the rate of 2°C per hour until midnight, at what time would the temperature be 8°C below zero? What would be the temperature at mid-night?

Initial temperature at noon = $+10^{\circ}$ C

Decrease rate = 2° C per hour

We want to reach = -8° C

Temperature drop needed = 10 - (-8) = 18°C

Time to drop 18° C at 2° C/hour = $18 \div 2 = 9$ hours

So, time =
$$12 \text{ noon} + 9 \text{ hours} = 9 \text{ PM}$$

Temperature at midnight:

Total drop =
$$2 \times 12 = 24$$
°C

So,
$$10 - 24 = -14$$
°C

Answer:

Time when temperature is -8°C: 9 PM

Temperature at midnight: -14°C

6. In a class test (+ 3) marks are given for every correct answer and (-2) marks are given for every incorrect answer and no marks for not attempting any question. (i) Radhika scored 20 marks. If she has got 12 correct answers, how many questions has she attempted incorrectly? (ii) Mohini scores – 5 marks in this test, though she has got 7 correct answers. How many questions has she attempted incorrectly?

Marks: +3 for correct, -2 for incorrect

(i) Radhika scored 20 marks, got 12 correct answers

Marks from correct answers = $12 \times 3 = 36$

Total score = 20

So, negative marks = 36 - 20 = 16

Incorrect answers = $16 \div 2 = 8$

Radhika attempted 8 questions incorrectly.

(ii) Mohini scored -5 marks, got 7 correct answers

Marks from correct answers = $7 \times 3 = 21$

Total score = -5

So, loss due to wrong = 21 - (-5) = 26

Incorrect answers = $26 \div 2 = 13$

Mohini attempted 13 questions incorrectly.

7. An elevator descends into a mine shaft at the rate of 6 m/min. If the descent starts from 10 m above the ground level, how long will it take to reach – 350 m.

Starts at +10 m

Ends at -350 m

Total distance = 10 + 350 = 360 m

Speed = 6 m/min

Time = $360 \div 6 = 60$ minutes

It will take 60 minutes (1 hour) to reach -350 m.