1 PATTERNS IN MATHEMATICS

1.1: What is Mathematics?

Q1. Can you think of other examples where mathematics helps us in our everyday lives?

Ans: Yes, examples include budgeting while shopping, setting alarms, calculating distances during travel, cooking with measurements, and understanding weather reports.

Q2. How has mathematics helped propel humanity forward?

Ans: Mathematics has enabled:

- Scientific experiments and discoveries
- Building infrastructure like bridges and buildings
- Technological advancements like mobiles and computers
- Space exploration
- Designing economic models and running democracies

1.2: Patterns in Numbers

1, 1, 1, 1, 1, 1, 1, ...	(All 1's)
1, 2, 3, 4, 5, 6, 7, ...	(Counting numbers)
1, 3, 5, 7, 9, 11, 13, ...	(Odd numbers)
2, 4, 6, 8, 10, 12, 14, ...	(Even numbers)
1, 3, 6, 10, 15, 21, 28, ...	(Triangular numbers)
1, 4, 9, 16, 25, 36, 49, ...	(Squares)
1, 8, 27, 64, 125, 216, ...	(Cubes)
1, 2, 3, 5, 8, 13, 21, ...	(Virahānka numbers)
1, 2, 4, 8, 16, 32, 64, ...	(Powers of 2)
1, 3, 9, 27, 81, 243, 729, ...	(Powers of 3)

Q1. Can you recognize the pattern in each of the sequences in Table 1?

Ans: Yes, the patterns are:

All 1s: Repeating 1

Counting Numbers: Add 1 each time

Odd Numbers: Add 2 starting from 1

Even Numbers: Add 2 starting from 2

Triangular Numbers: Add consecutive numbers

Squares: n^2

Cubes: n^3

Virahānka Numbers: Fibonacci series

Powers of 2: Multiply by 2

Powers of 3: Multiply by 3

Q2. Rewrite each sequence with the next 3 terms and define the rule.

Ans:

All 1s → 1, 1, 1, 1, 1, 1, 1, 1, 1 (Rule: Repeat 1)

Counting → 1 to 7, 8, 9, 10 (Add 1)

Odd → 1 to 13, 15, 17, 19 (Add 2)

Even → 2 to 14, 16, 18, 20 (Add 2)

Triangular → 28, 36, 45, 55 (Add 8, then 9, ...)

Squares → 49, 64, 81, 100 (n^2)

Cubes → 216, 343, 512, 729 (n^3)

Virahānka → 21, 34, 55, 89 (Add last 2 terms)

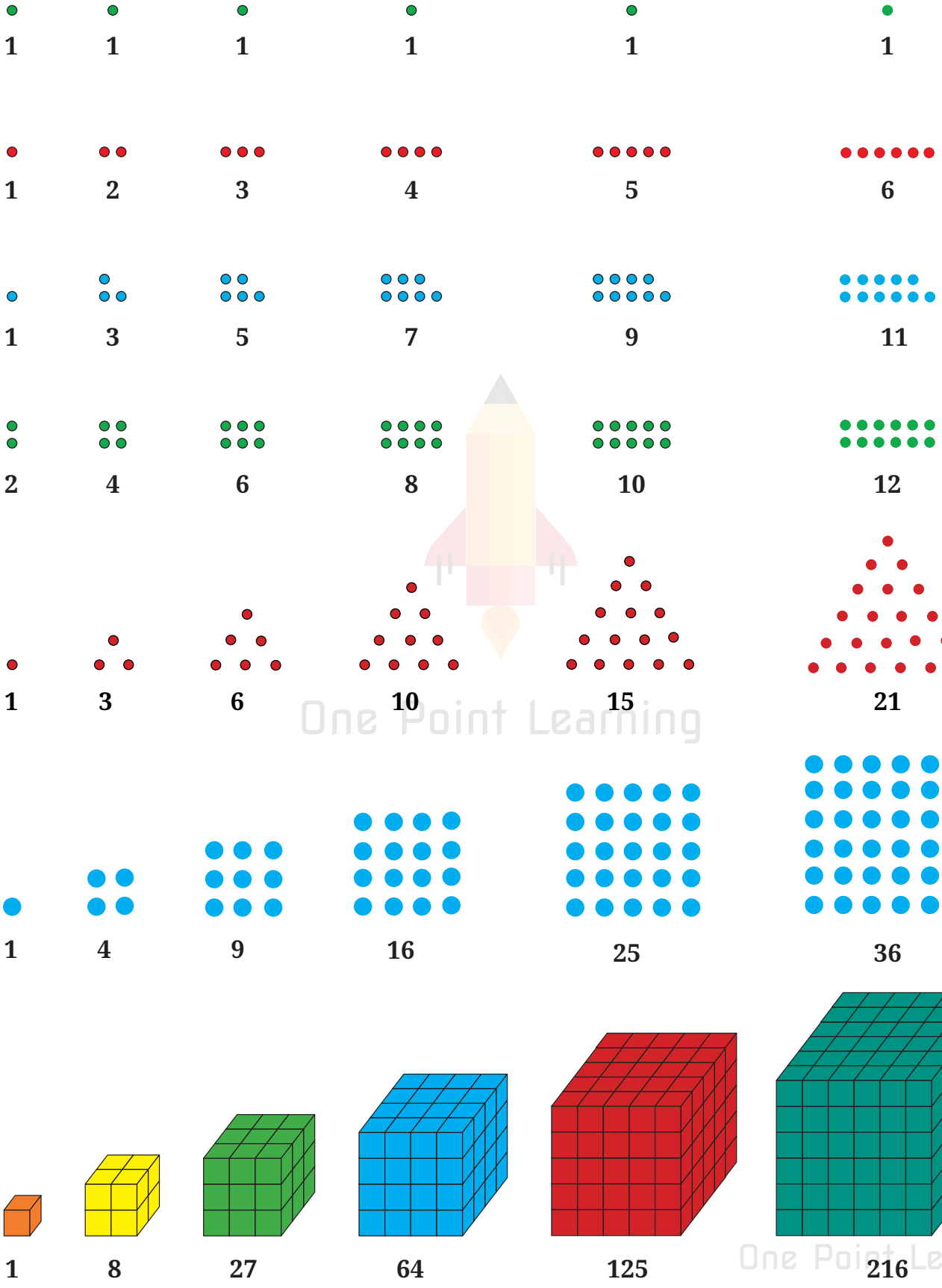
Powers of 2 → 64, 128, 256, 512 (×2)

Powers of 3 → 729, 2187, 6561, 19683 (×3)

1.3: Visualising Number Sequences

Q1. Copy the pictorial representations of the number sequences in Table 2 in your notebook, and draw the next picture for each sequence!

Ans:



Q2. Why are 1, 3, 6, 10, 15, … called triangular numbers? Why are 1, 4, 9, 16, 25, … called square numbers or squares? Why are 1, 8, 27, 64, 125, … called cubes?

Ans:

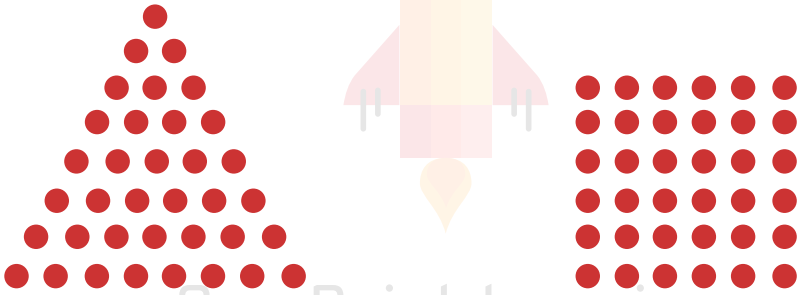
Triangular: can be represented by triangular arrangements of dots.

Square: can be represented by square arrangements of dots.

Cube: Represent 3D cubes, $n \times n \times n$.

Q3. You will have noticed that 36 is both a triangular number and a square number! That is, 36 dots can be arranged perfectly both in a triangle and in a square. Make pictures in your notebook illustrating this! This shows that the same number can be represented differently, and play different roles, depending on the context. Try representing some other numbers pictorially in different ways!

Ans: Draw a triangle with 8 rows (triangular number) and a 6×6 grid (square number).

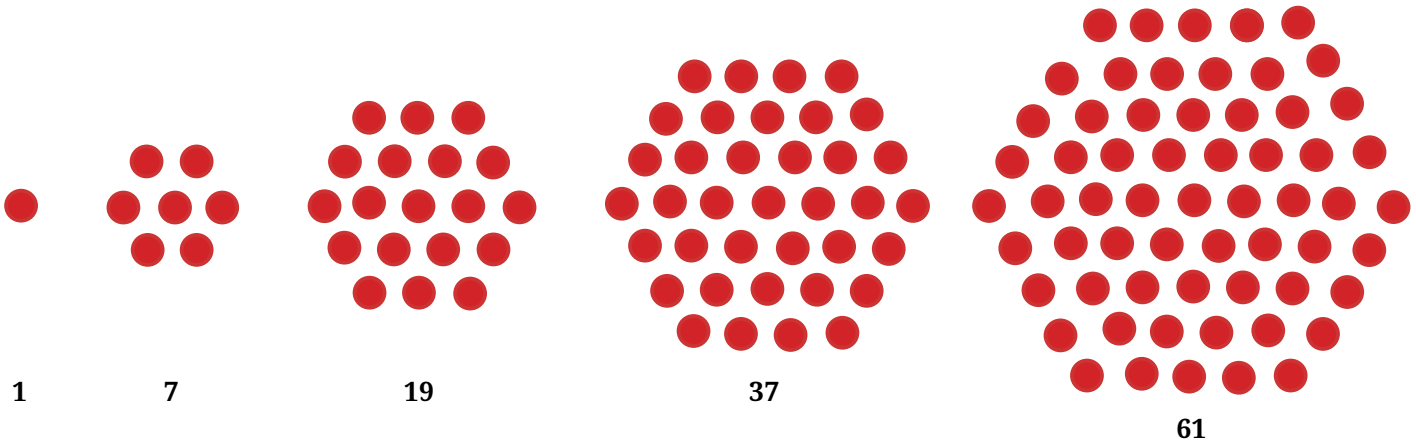


16

15

Q4. What is the next number in the hexagonal sequence: 1, 7, 19, 37?

Ans: 61 (Differences: +6, +12, +18 → next +24)



1.4: Relations Among Number Sequences

Q1. Why does adding odd numbers give square numbers?

Ans: Because the sum of first n odd numbers is n^2. Visualised as building a square layer by layer.

Sum of first 10 odd numbers: $10^2=100$

Sum of first 100 odd numbers: $100^2=10,000$

Q2. What is the value of 1 + 2 + 3 + ... + 100 + 99 + ... + 1?

A: $100^2=10,000$

Q3. Adding "All 1s" sequence:

Up: 1, 2, 3, 4, ...

Up and down: 1, 3, 5, 7, ...

Q4. Adding counting numbers up:

1, 3, 6, 10, 15 → Triangular Numbers

Q5. What do you get from 1 + 3, 3 + 6, 6 + 10, etc.?

A: 4, 9, 16, 25 → Square numbers

Q6. Adding powers of 2 and then adding 1:

1 → 2, 1+2=3 → 4, 1+2+4=7 → 8 → Pattern: One less than the next power of 2

Q7. Multiply triangular numbers by 6 and add 1 → 7, 19, 37 → we get Hexagonal numbers

Q8. Sum of hexagonal numbers

1, 1 + 7 = 8, 1 + 7 + 19 = 27, 1 + 7 + 19 + 37 = 64, ...

Clearly, we get 1, 8, 27, 64, 125 .... we get square numbers